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14MDE11

First Semester M.Tech. Degree Examination, Dec.2017/Jan.2018

Applied Mathematics

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

- 1 a. Write short notes on the following:
- i) Inherent error ii) Truncation error iii) Absolute error
 iv) Relative error iv) Percentage error (10 Marks)
- b. Prove that the differential equation of falling body like a parachutist is in the standard form is $\frac{dv}{dt} + \left(\frac{c}{m}\right)v = g$. And also prove its solution is $V = \frac{mg}{c} \left[1 - e^{-\left(\frac{c}{m}\right)t} \right]$ with initial condition $v = 0$ at $t = 0$. (10 Marks)

- 2 a. Derive the iterative formula to find the root of equation $f(x) = 0$ by Newton-Raphson method. Hence find the root of $e^x \sin x - 1 = 0$. (10 Marks)
- b. Perform three iterations of the Muller method to find the smallest positive root of the equation $x^3 - 5x + 1 = 0$. (10 Marks)

- 3 a. Find the approximate value of $\int_0^1 \frac{dx}{1+x}$ using (i) Trapezoidal rule and (ii) Simpson's rule. Obtain a bound for the errors. (10 Marks)

- b. Find the first and second derivative of $f(x)$ at $x = 1.2$ for the following data:

x	1.1	1.2	1.3	1.4	1.5
f(x)	1.3356	1.5095	1.6983	1.9043	2.1293

(10 Marks)

- 4 a. Solve the equations $-2x - y - 3z = 3$, $2x - 3y + z = -13$, $2x - 3z = -11$ using Cramer's rule. (06 Marks)
- b. Apply Gauss Jordan method to solve the equations $x + y + z = 9$, $2x - 3y + 4z = 13$, $3x + 4y + 5z = 40$. (07 Marks)
- c. Solve the system of equations $x + 2y + 3z = 5$, $2x + 8y + 22z = 6$ and $3x + 22y + 82z = -10$ using Cholesky method. (07 Marks)

- 5 a. Determine the inverse of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{pmatrix}$ using the partition method. Hence find the solution of the system of equations $x + y + z = 1$, $4x + 3y - z = 6$, $3x + 5y + 3z = 4$. (10 Marks)

- b. Using the Jacobi method find all the eigen values and the corresponding eigen vectors of the

matrix, $A = \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$.

(10 Marks)

- 6 a. Use Householder's method to reduce the given matrix A into the tridiagonal form

$$A = \begin{pmatrix} 4 & -1 & -2 & 2 \\ -1 & 4 & -1 & -2 \\ -2 & -1 & 4 & -1 \\ 2 & -2 & -1 & 4 \end{pmatrix}$$

(10 Marks)

- b. Find all the eigen values of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 2 \end{pmatrix}$ using the Rutishauser method. Iterate till the elements of the lower triangular part are less than 0.05 magnitude. (10 Marks)

- 7 a. Perform five iterations of the bisection method to obtain a root of the equation $\cos x - xe^x = 0$. (06 Marks)

- b. By using Regula-Falsi method, find the root of $x \log_{10} x = 1.2$ lies between 2 and 3. (07 Marks)

- c. Find the largest eigen value and the corresponding eigen vector by power method for $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ taking initial approximation $(1 \ 1 \ 1)^T$. (07 Marks)

- 8 a. Find a least square solution of $AX = b$ for

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$b = \begin{pmatrix} -3 \\ -1 \\ 0 \\ 2 \\ 5 \\ 1 \end{pmatrix}$$

(10 Marks)

- b. Let π be the plane in \mathbb{R}^3 spanned by vectors $(1, 2, 2)$ and $(-1, 0, 2)$.

(i) Find an orthonormal basis for π .

(ii) Extend it to an orthonormal basis of \mathbb{R}^3 . (10 Marks)
